

Topic I Metric Spaces

Many of the arguments you have seen in several variable calculus are almost identical to the corresponding arguments in one variable calculus, especially arguments concerning convergence and continuity. The reason is that the notions of convergence and continuity can be formulated in terms of distance, and that the notion of distance between numbers that you need in the one variable theory, is very similar to the notion of distance between points or vectors that you need in the theory of functions of severable variables.

In more advanced mathematics, we need to find the distance between more complicated objects than numbers and vectors, e.g. between sequences, sets and functions. These new notions of distance leads to new notions of convergence and continuity, and these again lead to new arguments suprisingly similar to those we have already seen in one and several variable calculus. The general theory that cover all these examples is the theory of metric spaces.

Topic II Differential Equations and Linear Algebra

Differential equations and linear algebra are the two crucial courses in undergraduate mathematics.

Differential equations are the fundamental tools that modern science and engineering use to model physical reality. The importance of differential equations to these disciplines cannot be overemphasized. A distinct subject in its own right, linear algebra is a part of mathematics concerned with the structure inherent in mathematical systems.

We will seek to understand the problem of existence and uniqueness of solutions to differential equations, expression and approximaion of such solutions, and understanding differential equations that model physical systems. We shall study these subjects together for three reasons: (1) The viewpoint of linear algebra is immensely helpful in uncovering the order underlying the topic of differential equations; it helps us understand the "why" and not just the "how" of our calculations; (2) Linear algebra is essential to the theory of differential equations; (3) Linear algebra is crucial to he computer approximations which are often the only way to solve the most challenging differential equations.